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THE LAMINAR, TURBULENT AND TRANSITIONAL FREE SURFACE FLOW OF NON-NEWTONIAN SUSPENSIONS

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Abstract: The free surface flow of high concentration non-Newtonian suspensions is becoming more prevalent in modern industry. The mining industry in particular is considering innovative tailings dewatering technologies such as high density thickeners, paste thickeners, and filtration to higher density. High concentration thickened tailings promotes both water recovery, and helps reduce embankment construction costs and reduce environmental risks. Dewatered tailings often travels some distance to the disposal site and are normally transported through pipe or open-top flumes or launders. But with increase in density, the tailings exhibits significant non-Newtonian behaviour and flow prediction of such material presents considerable challenges, when compared to that for Newtonian fluids. Where the terrain permits, an open channel is an economical alternative to a pipeline. However, unlike pipe flow, the open channel flow of non-Newtonian fluids such as thickened tailings has not received much attention and only limited investigations have been conducted to study open channel flow. However, in many studies, the pipe flow paradigm is adopted to hydraulically design open channels. The objective of this paper is to present a set of models as a basis to design open channel flow of thickened tailings material in the laminar and turbulent regimes. In particular, a Reynolds number approach to transition prediction is based upon true wall shear – rather than bulk shear as is more often the case. It is shown that for yield stress materials this change in approach has a significant effect. The design approach is validated using new copper tailings and kaolin suspension data, as well as published data. Furthermore, a criterion to discriminate the onset of transitional flow in open channels is developed and compared with these experimental data.

KEY WORDS: Open Channel, Non-Newtonian, Thickened Tailings

1 INTRODUCTION

To improve water recovery and reduce the environmental risks associated with tailings disposal, the mining industry is considering modern tailings dewatering technologies such as high-density thickeners, paste thickeners, and filtration to higher density. Hyper-concentrated thickened tailings, in addition to water recovery and environmental issues helps reduce embankment construction costs. Dewatered tailings normally travels some distance to the disposal site and are normally transported through pipe or open-top flumes or launders. But with increase in density, the tailings exhibits significant non-Newtonian behaviour and flow prediction of such material becomes challenging, when compared to Newtonian fluids (Gawu & Fourie 2004; Slatter et al. 2011).

If the terrain permits, an open channel can be an economical alternative to a pipeline. However, unlike pipe flow, despite recent research, open channel flow of non-Newtonian fluids such as thickened tailings has not received much attention and only limited investigations have been conducted to study open channel flow (Kozicki & Tiu 1967; Wilson 1991; Coussot 1994; Haldenwang & Slatter 2006; Fitton 2007; Pirouz et al. 2013; Burger 2014). In many studies, the pipe flow paradigm is adopted to hydraulically design open channel by using the pipe diameter as four times the hydraulic radius. The objective of this paper is to present a set of models as a basis to design open channel flow of thickened tailings material in the laminar and turbulent regimes. The design approach is validated by using the experimental measurement. Moreover, a criterion to discriminate the onset of transitional flow in open channels is developed and compared with the experimental data.

2 FLOW REGIMES MODELLING

Classical fluid mechanics experiment and modelling has shown that flow behaviour can be characterised in two main regimes – laminar and turbulent. Transition from the one regime to the other is not direct, and a transitional flow regime can be identified. These typical flow behaviours are immediately evident from the experimental results (see for example the results portrayed in Figure 4) and are analysed and modelled in this section.

2.1 LAMINAR FLOW

For the laminar flow of non-Newtonian material through a channel, Kozicki and Tiu (1967) developed an analytical method. The method involves the use of two geometric parameters, a and b, to describe the channel cross section. The prediction obtained from this model showed poor agreement with the experimental results of yield pseudo-plastic non-Newtonian material (Haldenwang 2003; Javadi et al. 2014). Most of the current models associated with open channel flow design, other than analytical method by Kozicki and Tie 1967, were derived from the pipe flow paradigm (Haldenwang et al. 2010; Burger et al. 2010; Haldenwang 2003). The predictions by these models are comparable with the experimental records with a reasonable accuracy.

In this study, for the laminar flow regime, a model is presented in order to evaluate the appropriate estimate of the wall shear stress (Slatter 2015; Javadi et al., 2015). To achieve this, the model incorporates the hydraulic and rheological properties of flow. The model adopts the flow concepts of two geometries – pipe flow and infinitely wide channel flow - to define a new formulation for the Reynolds number.

By substituting the hydraulic radius and channel slope S_o in the Darcy–Weisbach equation, the following is obtained for open channel flow of arbitrary geometry (Abulnaga 2002):

$$S_o = \frac{16}{Re} \frac{1}{R_h} \frac{V^2}{g} \tag{1}$$

Where Re is Reynolds number, R_h is hydraulic radius, V is average velocity and g is gravitational acceleration. On the basis that Reynolds number is defined as the ratio of inertial forces to viscous forces and by adopting the true wall shear stress of sheet flow and hydraulic radius (R_h), one can develop a generalised formulation for Reynolds number as follows:

$$Re_{New} = \frac{8\rho V^2}{\tau_y + K\left(\frac{3V}{R_h}\left(\frac{2n^*+1}{3n^*}\right)\right)^n} \tag{2}$$

$$n^* = \frac{dln(\tau_w)}{dln\left(\frac{3V}{R_h}\right)} \tag{3}$$

Where τ_y, K, n are the Herschel-Bulkley rheological model parameters. This model, in comparison with Haldenwang and Slatter (2006), considers true shear rate rather than bulk shear rate in order to accurately estimate the friction caused by viscous resistance at the flow boundary. The details of the model development were presented in Javadi et al. (2015).

2.2 TRANSITIONAL FLOW

Given the fact that almost all suspensions – including thickened tailings – tend to deposit if insufficient turbulence is available, it is often desired to design a system to transport thickened tailings material in the transitional range from laminar to turbulent flow. However, the laminar regime can still be a viable flow regime for non-settling slurry transported short distances. For non-Newtonian fluid containing suspensions, the arbitrary velocity fluctuations caused by transitional flow can stir up the settling particles. Hence, it is of critical importance to accurately discriminate the onset of transition for such flows.

For non-Newtonian flow in a pipe, many attempts have been made to distinguish the transition point from laminar to turbulent. By analogy to Newtonian fluid transition, some works supposed the transition occurs about Re =2,100, and manipulated the transitional criterion (Hanks 1963; Govier & Aziz 1972; Metzner & Reed 1955; Slatter 2011; Griffiths 2012). Slatter and Wasp (2000) compared several models and found that all approaches are not able to define the critical velocity over all ranges of Hedstrom number, therefore they developed three empirical models for three different ranges of Hedstrom number (Slatter & Wasp 2000). Their approaches were further analytically investigated by Wilson and Thomas (Wilson & Thomas 2006). In channel flows, limited progress has been made in comparison with pipe flow, and most of the approaches are developed either from the pipe flow criterion or experimental results obtained in flume tests (Haldenwang & Slatter 2006; Haldenwang et al. 2010; Slatter et al. 2011).

In this paper an analytical criterion for the laminar/turbulent transition is presented by introducing a stability number. It is assumed that at the limit of the transitional zone, the laminar shear velocity (U_L^*) approaches the turbulent regime shear velocity (U_T^*) and the stability number is defined as the ratio of these two parameters (Javadi et al., 2015). Therefore, at regime change, the stability number (k) approaches unity and defined as follows:

$$k = \sqrt{S_o} - 0.14\sqrt{Re} - \ln\left(\frac{4\rho R_h^2 g^{0.5}}{3.13\,\mu_{eff}}\right) \tag{4}$$

Where, μ_{eff} is the apparent viscosity, defined here as the slope of the tangent line to the rheogram curve at a shear rate of 100 1/s.

The magnitude of k indicates the flow regimes and can vary from infinitely negative for extremely turbulent to infinitely positive for extreme laminar flows.

For $k<1$ Transitional/turbulent flow.
$k>1$ Laminar flow.
$k=1$ Critical flow.

2.3 TURBULENT FLOW

Non-Newtonian turbulent flow is believed to be fundamentally similar to Newtonian turbulent flow, since it is an inertia dominated process. In the other words, the existence of a thin viscous sub-layer at the flow boundary, a logarithmic velocity profile above the viscous sub-layer, eddy flow formation and dissipation are all occur in both Newtonian and Non-Newtonian fluid. To model the turbulent flow, nevertheless considerable idealization is employed, even for a Newtonian fluid in circular geometries. In channel flow, due to the more complex boundary geometry, analytical evaluation of flow behaviour is fraught with more complication in comparison with pipe flow (Wilson & Thomas 2006; Fitton 2007). This has led to semi-empirical models most of which are derived from the pipe flow paradigm with empirical constants obtained from experimental data (Wilson 1991; Haldenwang 2003; Haldenwang et al. 2010).

However, as far as can be ascertained, one of the turbulent models for non-Newtonian slurry which is being widely used for engineering purpose is the Wilson and Thomas (1985) smooth pipe model. Their model includes corrections to account for the thickening of the viscous sub-layer caused by the rheology of the material and has shown good agreement with experimental results. Their work was advanced and extended to rough boundaries (Thomas & Wilson 2007). Assuming that the same pipe sub-layer thickening occurs at the channel boundary for thickened tailings, in our work we adopted this model in order to define a design approach in the turbulent regime as follows:

$$\frac{V}{U^*} = 2.5\ln\left(\frac{4\rho U^* R_h}{\mu_{eff}}\right) - 2.5\ln(\alpha - 1) + 11.6\,(\alpha - 1) - \Omega \quad (5)$$

Where:

$$\Omega = -2.5\ln\left(1 - \frac{\tau_y}{\tau_w}\right) - 2.5\frac{\tau_y}{\tau_w}\left(1 - \frac{\tau_y}{2\tau_w}\right) \quad (6)$$

and α is the ratio of area of a non-Newtonian fluid rheogram to that of an equivalent Newtonian fluid at the same wall shear stress. One can express α for a Herschel–Bulkley material as:

$$\alpha = \frac{2(n+1)\tau_y + 2\,(\tau_w - \tau_y)}{(n+1)\tau_w} \quad (7)$$

3 EXPERIMENTAL RESULTS AND DISCUSSION

To validate and evaluate the new models, two sets data were used. The first data set is from a newly compiled experimental data base. These data were measured and recorded at RMIT University using a 4.8m long tilting flume. This flume can be tilted at various angles of up to 9° from the horizontal. The width of this rectangular flume is 100 mm. The second data set used is from the test work carried out at the Flow Process Research Centre at the Cape Peninsula University of Technology (Haldenwang 2003; Haldenwang & Slatter 2006). The fluid materials are kaolin and copper tailings suspensions. These materials exhibit Hershel–Bulkley rheological behaviour and the rheological parameters are summarized in Table 1.

Table 1 Summary of material tested

Suspension	Cv (%)	τ_y (Pa)	k $(Pa\,s^n)$	n
Kaolin (Haldenwang-Slatter 2006)	10	21.311	0.428	0.468
Kaolin (Haldenwang-Slatter 2006)	7.1	9.431	0.625	0.388
Kaolin (Haldenwang-Slatter 2006)	6	6.840	0.148	0.517
Kaolin	3	1.38	0.118	0.503
Copper Tailings + Kaolin Mixture	4	1.26	0.25	0.417

To evaluate the new model, predictions obtained from the model are compared with the experimental data. In addition, the new model accuracy is compared with the previously published models in both the laminar and turbulent regimes. The followings steps are used to predict the flow behaviour in the laminar and turbulent flow by each model:

1. Assume an initial value for the depth in the channel (h)
2. Calculate the cross sectional area (A) and wetted perimeter of flow (P)
3. Calculate hydraulic radius (Rh)
4. Calculate average velocity (V) based on the cross sectional area and given flow rate (Q/A)
5. Calculate Re number (using Reynolds number formulation corresponding to the each model)
6. Calculate friction factor f (using corresponding formulation to the each model based on the flow range)
7. Calculate V using equation (1)
8. Adjust the initially assumed depth value until the two values for V calculated in steps 4 and 7 equate.
9. At this point, the hydraulic radius, friction factor and average velocity are the model prediction

In Figures 1 and 2, the predicted average velocity for the new model (Equations 2 and 3) in the laminar regime is illustrated. To evaluate the new model, the predictions were compared with the measured velocity. It is quite evident that the agreement is very good for copper tailings and kaolin and suspensions at different concentrations in the laminar regime. The absolute error of our predictions, with few exceptions, does not exceed 10%.

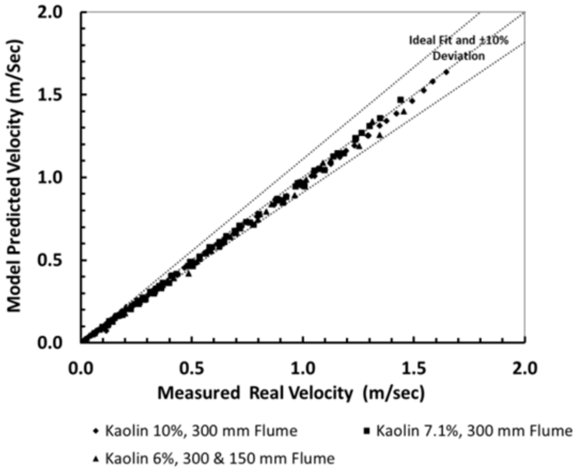

Figure 1 Predicted and experimental average velocity in the laminar regime for kaolin at varying solid concentrations

The new model was evaluated further against previously defined models that were developed based on the pipe flow and open channel flow paradigm. These comparisons are presented in Figures 3 and 4, indicating that the best predictions of the measured average velocity are obtained by the new model. However, the predictions of the other models, except the sheet flow models, are comparable with the new model.

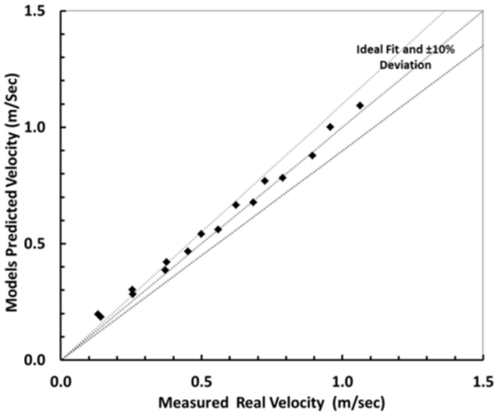

Figure 2 Predicted and experimental average velocity in the laminar regime for copper tailings and kaolin suspension

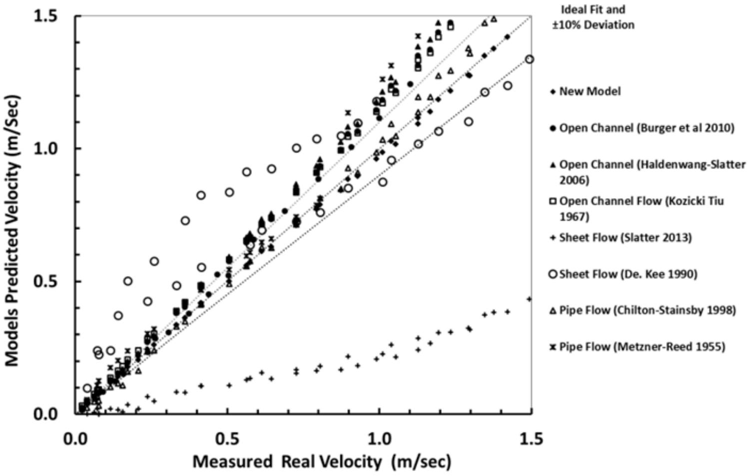

Figure 3 Predicted and experimental average velocity for Kaolin suspension in the laminar regime (10% volumetric concentration)

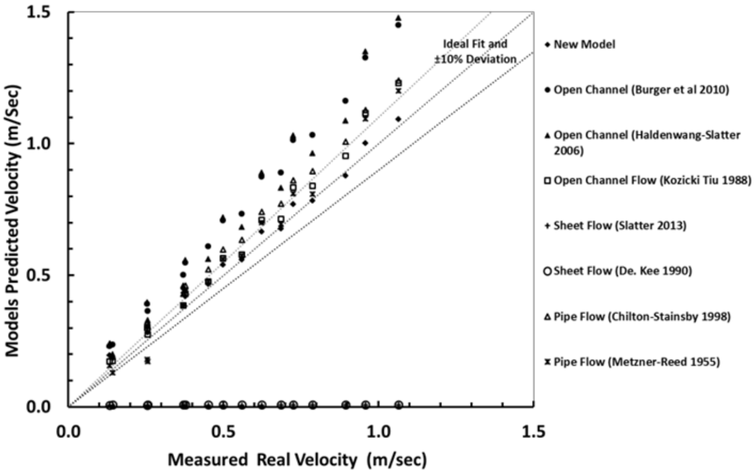

Figure 4 Predicted and experimental average velocity copper tailings and kaolin suspension

Figures 5 and 6 illustrate the transition point discriminated by the stability number *k* that is presented in this paper for kaolin suspensions flowing at slopes of 4 and 5°. The onset of transitional flow occurs at the deflection point of plots of wall shear stress against bulk shear rate on the logarithmic axis (Wilson 1991; Haldenwang 2003). The *k* value for the measured data is calculated from Equation 4. For each set of data there is a horizontal line indicating the *k*=1 and the locus of the measured points compared to this line indicates the flow regime. Flow conditions of the points located above the line *k*=1 (e.g. *k*=0.1 in Figure 5 and *k*=0.799 in Figure 6) are in the transitional/turbulent range while the other points are in the laminar region. Given that the stability number *k*=1 represents the transition from laminar to turbulent, it is clear that the stability parameter quite accurately predicts the onset of transition for the data obtained from the experimental results. However, this approach needs to be validated with more data.

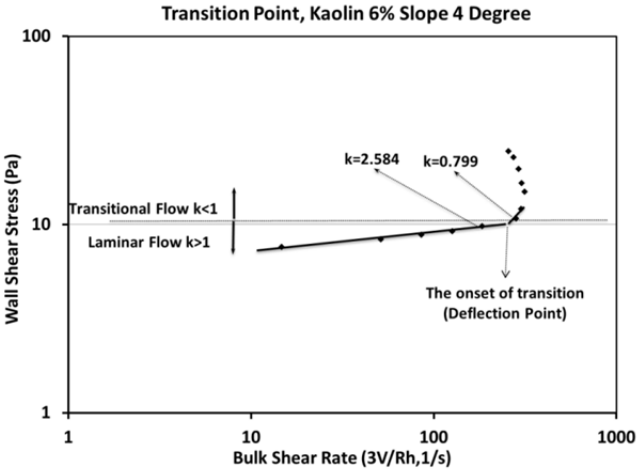

Figure 5 Locus of transition point (Kaolin 6% slope 4°)

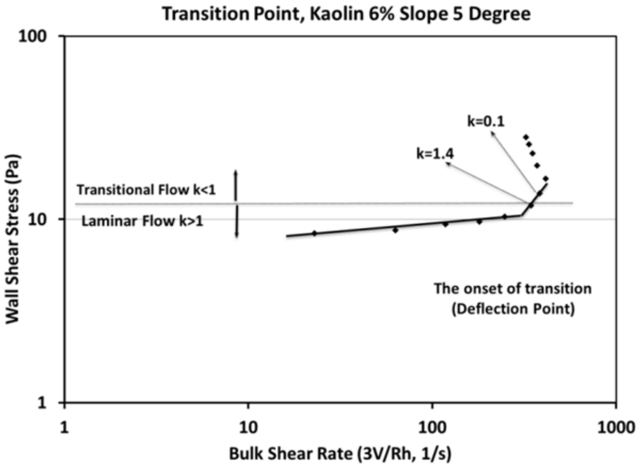

Figure 6 Locus of transition point (Kaolin 6% slope 5°)

Figure 7 shows the plots for copper and kaolin suspension in 100 mm rectangular channel. The plots correspond to the average velocity predicted by the modified Wilson–Thomas model and the models reported in the literature. The models are Burger et al. (2015), Wilson-Thomas (2006), Haldenwang-Slatter (2003), Chilton-Stainsby (1998) Darby (1992), Wilson-Thomas (1985) and Dodge-Metzner (1959). As observed for the turbulent flow of copper tailings and kaolin suspension in the 100 mm channel, the plots show a good agreements between the predicted velocities obtained from the new models and some of the previous model with the measured velocities. Burger et al. 2015 and Chilton-Stainsby models tend to over predict the average velocity with a deviation of over 20% whereas a clear velocity underestimation from Darby model is observed.

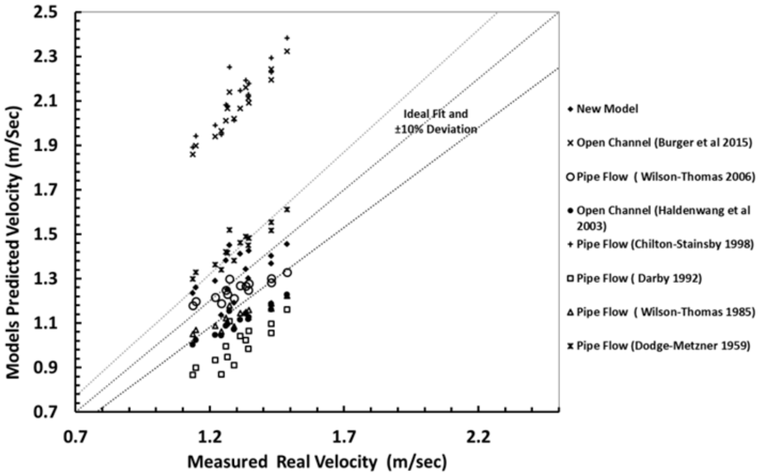

Figure 7 Turbulent flow velocity prediction (Copper tailings and Kaolin suspension)

It can be seen from the plots of average velocity that the new models give a quite tighter fit to the experimental data. However, the comparison between the models and the experimental results is not an objective. To rank the models accuracy, Burger et al. (2015) adopted an objective measure which was recommended by Lazarus and Nielson (1978). This method employs the log standard error (LSE) in one of the flow parameters (e.g. average velocity, friction factor and hydraulic radius). In spite of simple usefulness, the comparison of the models is only limited to one parameter (often average velocity). In other words, it might be questioned that how accurate the models are in terms of the friction factor and hydraulic radius prediction.

To consider the error made in all flow parameters, in this paper a combined error in the flow parameters obtained by the model prediction is adopted. It is obvious that, based on equation (1), the channel slope is a function of the flow parameters. On this basis, the uncertainty of channel slope incurred by the independent parameters' error can be determined by the root-sum-square (RSS) expression (Tang et al., 2012). The lower RSS, the better is the model fit to the experimental data. RSS is given as:

$$S_o = F(f, R_h, V) \tag{6}$$

$$RSS = \delta S_o = \sqrt{(\frac{\partial S_o}{\partial f}\delta f)^2 + (\frac{\partial S_o}{\partial R_h}\delta R_h)^2 + (\frac{\partial S_o}{\partial V}\delta V)^2} \tag{7}$$

Where δf, δR_h and δV are mean root squared errors in friction factor, hydraulic radius and average velocity respectively. The errors are deviations of the prediction from the measured data. Tables 2 and 3 summarize the RSS values for flow of various suspensions in the laminar and turbulent regimes. In the laminar regime, the RSS values for the new models and the five models suggest that the new model along with Kozicki Tiu 1988 model gives the best fit to the measured data. It is worth mentioning that these two models employ an extra term of fluid index n^* to evaluate the Reynolds number and hence the higher accuracy of these models might be attributed to using n^*. In the turbulent range, the new model which was based on a modified Wilson-Thomas model gives a lower RSS values, indicating the best fit from the new model. Dodge-Metzner and Wilson-Thomas prediction are also comparable to the new model.

Table 2 RSS values in the laminar regimes

Model	Kaolin 10% * Slope: 8.7%, 7%, 5.2%, 3.5%	Kaolin 7.5% Slope: 5%, 6%, 7%, 8%	Kaolin + Copper Tailings 10% Slope: 5%, 6%, 7%, 8%
Kozicki-Tiu 1988	0.0062	0.0334	0.0141
Metzner-Reed 1955	0.0258	0.0359	0.0533
Chilton-Stainsby 1998	0.0013	0.0169	0.0276
Haldenwang-Slatter 2006	0.0065	0.0295	0.0444
Burger et al 2010	0.0055	0.0281	0.0466
New Model	0.0009	0.0165	0.0158

* Haldenwang-Slatter 2006 data

Table 3 RSS values in the turbulent regimes

Model	Kaolin 7.5% Slope: 5%, 6%, 7%, 8%	Kaolin + Copper Tailings 10% Slope: 5%, 6%, 7%, 8%
Dodge-Metzner 1959	0.0133	0.0209
Wilson Thomas 2006	0.0202	0.0202
Wilson Thomas 1985	0.0167	0.0214
Chilton-Stainsby 1998	0.0548	0.0778
Burger et al 2015	0.0699	0.0815

Darby 1992	0.0704	0.0783
Haldenwang-Slatter 2003	0.0604	0.0562
New Model	0.0102	0.0101

4 CONCLUSION

A set of semi-theoretical models to predict the laminar, transitional and turbulent flow of non-Newtonian material have been presented in this paper. These models were found to agree closely with the experimental flume data for kaolin and copper tailings suspensions.

The prediction accuracy of the new models was compared against the previously published models. It was shown from RSS values that the new models were either better or equivalent to the published models. In the laminar regime, the new models along with Kozicki-Tiu 1988 models give the best fit to the experimental records that it is believed to be attributed to the usage of an extra term of fluid index n^* to evaluate the Reynolds number by these two models. However, it was found that the predictions of the other models are also comparable to these models.

In the turbulent range, the new model which was derived from the Wilson-Thomas model gives a tighter fit to the experimental data. This is shown in the lower RSS values, indicating that the best fit is from the new model. The Dodge-Metzner and Wilson-Thomas predictions are, nevertheless, comparable to the new model.

A new analysis to establish the onset of transitional flow has been presented and validated using the experimental data. This approach accurately predicts the laminar/turbulent transition.

However, further confirmation of the validity of the models presented is required using a wider range of real tailings flow data.

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